Julio A. Gonzalo
Facultad de Ciencias
Universidad Atónoma de Madrid
Cantoblanco 28049 Madrid Spain
email: julio.gonzalo@uam.es
(24-10-2000)

Properly interpreted data from nearby galaxies ($z \simeq 0.01$) lead to $\Omega \simeq 0.082$. Data from farther away galaxies ($z \simeq 1$) with type Ia supernovae to $\Omega = 0.153$. Data to be expected from very high redshifted galaxies ($z \simeq 10.1$) to $\Omega = 0.500$. And actual data from the CBR, emitted at the time at which the universe became transparent ($z \simeq 1422$) to $\Omega \simeq 0.992$. All these data are simultaneously consistent with the standard big-bang picture (no inflation), in which Ω is time dependent and it is given by $\Omega(y) = 1/\cosh^2(y)$, being $y \equiv \sinh^{-1}(T_+/T)^{1/2}$

It is well known [1,2] that Einstein´s cosmological equations for **an open universe** can be written as

$$\overset{\circ}{R} = R^{-1/2}\left\{(8\pi/3)G\rho R^3 + |k|\, c^2 R\right\}^{1/2} \tag{1}$$

where $R(t)$ is the scale factor or radius of the universe, $\overset{\circ}{R}(t)$ its time derivative, $\rho(t)$ the mass density per unit volume, being $\rho(t) = \rho_m(t) + \rho_r(t)$ the sum of matter (ρ_m) and radiation (ρ_r) mass densities, and $k < 0$ the spatial curvature.

In the present matter dominated epoch $\rho_m >> \rho_r$ and $\rho \sim R^{-3}$. Then we take $\rho R^3 \sim const$ in the right hand side of Eq.1 and define

$$(8\pi/3)G\rho_+ R_+^3 = |k|\, c^2 R_+ \tag{2}$$

to write the integral as

$$\int dt = \int \frac{R^{1/2}}{\left\{(8\pi/3)\rho_+ R_+^3 + |k|\, c^2 R\right\}^{1/2}} dR \tag{3}$$

where $(8\pi/3)G\rho_+ R_+^3 = |k|\, c^2 R_+ = a^2$ is a constant. Using the change of variable $x^2 = |k|\, c^2 R$ the integral in the right hand side of Eq.3 is transformed to

$$\int \frac{x^2}{\{a^2 + x^2\}} dx,$$

which is found in tables, resulting in

$$t = \frac{R_+}{|k|^{1/2} c}\left\{\sinh(y)\cosh(y) - y\right\}, \tag{4}$$

where $(R/R_+)^{1/2} \equiv \sinh(y)$ has been used, i.e.

$$R = R_+ \sinh^2(y) \tag{5}$$

Eqs.4 and 5 give parametrically, in terms of y, the **cosmic time** $t(y)$ and the **cosmic radius** $R(y)$. Using Eqs.4 and 5 expressions for the Hubble parameter $H \equiv \overset{\circ}{R}/R$ and the dimensionless cosmic parameters (Ht) and $\Omega = (\rho/\rho_c)$ (where $\rho_c = 3H^2/8\pi G$ is the critical density), all of them time dependent, can be easily obtained. In particular

$$H = (c\,|k|^{1/2}/R_+)\cosh(y)/\sinh^3(y) \tag{6}$$

resulting in

$$(Ht) = \{\sinh(y)\cosh(y) - y\}\cosh(y)/\sinh^3(y) \geq 2/3 \tag{7}$$

$$\Omega = 1/\cosh^2(y) \leq 1 \tag{8}$$

Improved recent determinations [3] of $H_0 \simeq 65 \pm 10 km sec^{-1} Mpc^{-1}$ (mainly from nearby galaxies) and [4] $t_0 \simeq (13.7 \pm 2) \times 10^9$ years (from the oldest stars in the Milky Way globular clusters) result in

$$H_0 t_0 \simeq 0.91 \text{ (dimensionless)} \tag{9}$$

which , by Eq.7, implies $y_0 \simeq 1.92$. Then, taking into account that the corresponding cosmic **equation of state** $RT \simeq$constant,

$$y_0 = \sinh^{-1}(R_0/R_+)^{1/2} = \sinh^{-1}(T_+/T_0)^{1/2} \simeq 1.92, \tag{10}$$

which allows one to get $T_+ \simeq 30.4K$, using the well known COBE data [5] for $T_0 \simeq 2.726 \pm 0.01K$, and, consequently, to evaluate cosmic quantities at any $R = R_0/(1+z)$ and $T = T_0(1+z)$, being z the redshift.

First let us determine z_+ corresponding to $T_+ = 30.4K$ (i.e. to R_+ defined by Eq.2). By means of Eq.10 we get

$$y_0 \simeq \sinh^{-1}\left(\frac{1+z_+}{1+0}\right) = 1.92 \longrightarrow z_+ = 10.1 \tag{11}$$

This implies $y_+ = \sinh^{-1}(1) = 0.881$ for $z_+ = 10.1$, which corresponds to a density parameter $\Omega_+ = 1/\cosh^2(y_+) = 1/2$.

We can evaluate the evolution of the expected value of the density parameter $\Omega(z)$ for (1) $z \simeq 0.01$, **nearby galaxies**, (2) $z \simeq 1$, relatively **distant galaxies** (accurately characterized [6] recently by means of Type Ia Supernovae), (3) $z \simeq 10.1$, a guess on the upper redshift for far away **protogalaxies**, and (4) $z \simeq 1422$, corresponding to the distance at which **cosmic background fluctuations** are originated, i.e. the distance at which the universe is becoming transparent and therefore $T_{af} \simeq 3880K$ (i.e. $z = (T_{af}/T_0) - 1 \simeq 1422$).

Then we have

(1) $z = 0.01$ (**nearby galaxies**)

$$y_z = \sinh^{-1}\left(\frac{1+z_+}{1+z}\right) \simeq 1.913 \longrightarrow \Omega = 1/\cosh^2(y_z) \simeq 0.083$$

(2) $z = 1$ (relatively **distant galaxies**)

$$y_z = \sinh^{-1}\left(\frac{1+z_+}{1+z}\right) \simeq 1.592 \longrightarrow \Omega = 1/\cosh^2(y_z) \simeq 0.152$$

(3) $z = 10.1$ (**guess** for **protogalaxies**)

$$y_z = \sinh^{-1}\left(\frac{1+z_+}{1+z}\right) \simeq 0.881 \longrightarrow \Omega = 1/\cosh^2(y_z) \simeq 0.500$$

(4) $z = 1422$ (**CBR**, **the very first light**)

$$y_z = \sinh^{-1}\left(\frac{1+z_+}{1+z}\right) \simeq 0.0882 \longrightarrow \Omega = 1/\cosh^2(y_z) \simeq 0.992$$

This means that the standard big-bang model (no inflation) can account **directly** and **simultaneously** for the various observed Ω values, from $\Omega \simeq 0.1$ for light coming from our immediate neighborhood, all the way to $\Omega \simeq 1$ for light coming from the moment at which the universe became transparent. Here, no **"missing mass"**. No need for **"inflation"** to explain the "missing mass". No need for a **non-vanishing cosmological constant** to account for the apparent redshift dependence of distant cosmic objects.

The recent reported findings of the Boomerang Collaboration [7] ($\Omega = 1.06 \pm 0.06$) and the Maxima Collaboration ($\Omega = 0.90 \pm 0.07$), obtained by means of the analysis of the power spectrum of spatial temperature fluctuations from spherical harmonic fits to their CMB maps, are in very good agreement with our estimate [9] ($\Omega = 0.992$) of the cosmic density parameter **at the time the CBR was emitted**.

---

[1] S. Weinberg, "Gravitational Cosmology" (Wiley and Sons: New York, 1972).

[2] P. J. E. Peebles, "Principles of Astrophysical Cosmology" (Princeton University Press: Princeton, N.J., 1993).

[3] W. L. Freedman et al.Nature, **371,** 757-762 (1994).

[4] M. Botte and C. J. Hogan, Nature, **376**, 399-402 (1995).

[5] J. C. Mather et al. Astroph. J. **429**, 439-444 (1994).

[6] arXiv: astro-ph/9906463 v4 / 1 Aug 1999.

[7] arXiv: astro-ph/0005004 v2 / 9 Jul 2000.

[8] arXiv: astro-ph/0005124 / 8 May 2000.

[9] N. Cereceda, G. Lifante and J. A. Gonzalo, Acta Cosmológica (Universitas Iaguellonica) Fasciculus XXIV-2 (1998).